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NUCLEAR PHYSICS FOR NUCLEAR FUSION

---Selective Resonant Tunneling in Light Nuclei Fusion---

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Abstract

The nuclear fusion data for deuteron-triton resonance near 100 keV are found to be consistent with the selective resonant tunneling model. The feature of this selective resonant tunneling is the selectivity. It selects not only the energy level, but also the damping rate (nuclear reaction rate). When the Coulomb barrier is thin and low, the resonance selects the fast reaction channel; however, when the Coulomb barrier is thick and high, the resonance selects the slow reaction channel. This mechanism might open an approach towards fusion energy with no strong nuclear radiation.

Keywords: Selective Resonant Tunneling; Deuteron-Triton Fusion; Suppression of Neutron Emission; Coulomb Barrier.

I. CORRECT AN UNEXPECTED JUMP IN D+T CROSS SECTION

Plasma physics developed rapidly in the past 40 years due to the research and development for the controlled nuclear fusion devices. Plasma physics and nuclear fusion have been so closely related that some journals and institutions are named by plasma and fusion together. However, the overwhelming dominance of plasma physics attracted most of the theoretical effort in magneto hydrodynamics and kinetics, in turbulence and chaos, in instabilities and transport........ Unfortunately, nuclear physics was almost neglected. The unawareness of the nuclear physics was so evident that a set of seemingly good data was totally ignored for more than three years. The National Nuclear Data Center at Brookhaven National Laboratory has been so kind to provide the nuclear data to the scientists in the whole world free of charge. A set of data for d+t fusion cross-section was cited there in 1996.[1] It clearly showed that other than the famous 100 keV resonance peak [2], there was an unexpected jump in cross section at low energy near 100 eV (Fig.1, pointed by an arrow), the cross-section jumped to about 5 barns there unexpectedly. We will show later that the selective resonant tunneling model does not allow to have such a jump in cross section. Indeed this jump was just a mistake. If this set of data were true; then, the Lawson criterion [3] should have been much more favorable than that in 1957. (For example, the

Spherical Torus in UK (START) would have reached break-even, if this unexpected jump in cross section would have been a real resonance). However, no one noticed this mistake until the physics of resonant tunneling was revisited recently.

II. SELECTIVE RESONANT TUNNELING MODEL

In most of the literatures, resonant tunneling of the Coulomb barrier for the nuclear reaction was treated as a two-step process. That is: tunneling first; then, decay. The tunneling probability was calculated in an oversimplified one-dimensional model [4], and the decay was assumed to be independent of the tunneling process. Nevertheless, this is not true in the case of the light nuclei fusion. In reality, when the wave function of the projectile penetrates the Coulomb barrier, it will reflect back and forth inside the nuclear well. This reflection inside the nuclear well is totally neglected in the one-dimensional model where the wave has no reflection as long as it penetrates through the barrier.(In the case of α-decay, the outgoing α-particle will have no reflection after penetrating the Coulomb barrier even if in 3-dimensional model [5]). Indeed this reflection is essential for the resonant penetration into the center of nuclear well through the Coulomb barrier. Secondary, the decay of the penetrating projectile will terminate the motion of bouncing back and forth inside the nuclear well. If nuclear reaction happens quickly; then, the wave function will have no time to bounce back and forth. That is: the short lifetime of the penetrating wave may not allow a resonant tunneling, because there will be no enough bounce motion to build-up the wave function in terms of constructive interference inside the nuclear well. In a word, the tunneling and the decay in the light nucleus fusion should be combined together as a selective process. Tunneling and decay are no longer independent.

It has been shown that an imaginary part of potential inside the nuclear well is a proper way to consider this lifetime effect on the resonant tunneling. A complex nuclear potential is proposed to describe this resonant tunneling effect for sub-barrier fusion in a 3-dimensional model for wide range of the energy of the projectile [6-8]. In that 3-dimensional calculation, instead of conventional phase shift, δ_o , we introduced a new pair of parameters: W_r and W_i , the real and the imaginary parts of the cotangent of phase shift, i.e.

$$\cot\delta_0 = W_r + iW_i \,, \tag{1}$$

Thus, the fusion cross section for s wave will have a simple expression as

$$\sigma_r^{(o)} = \frac{\pi}{k^2}\frac{(-4W_i)}{W_r^2 + (W_i - 1)^2}\,. \tag{2}$$

This equation (2) expresses the resonant feature without invoking any Taylor expansion. When W_r=0, the cross section reaches the resonance peak. On the other hand, W_i determines the height and width of the resonance peak. Hence, we may call W_r the resonance function, and W_i the damping function. W_r and W_i may be expressed as the function of two other parameters: U_{1r} and U_{1i}, i.e. the real and the imaginary parts of the nuclear potential [8]

$$W_r = \theta^2 \left\{ \frac{a_c}{a} \frac{z_r \sin(2z_r) + z_i \sinh(2z_i)}{2[\sin^2(z_r) + \sinh^2(z_i)]} - 2\left[\ln\left(\frac{2a}{a_c}\right) + 2C + h(ka_c) \right] \right\}, \tag{3}$$

$$W_i = \theta^2 \left\{ \frac{a_c}{a} \frac{z_i \sin(2z_r) - z_r \sinh(2z_i)}{2[\sin^2(z_r) + \sinh^2(z_i)]} \right\}. \tag{4}$$

Here $1/\theta^2$ is the famous Gamow penetration factor,

$$\theta^2 = \frac{1}{2\pi}\left[\exp\left(\frac{2\pi}{ka_c}\right) - 1 \right]. \tag{5}$$

It is a function of incident energy E only, because $k^2 = (2\mu/\hbar^2)E$, and $a_c = \hbar^2/(Z_1 Z_2 \mu e^2)$ is a constant(the Coulomb unit of length). Here μ is the reduced mass, Z_1 and Z_2 are the charge number for the colliding nuclei, respectively; e is the charge unit of electricity, $\hbar$ is the Planck constant divided by 2π. A complex number z is defined as

$$z = k_1 a \equiv k_{1r} a + i k_{1i} a \equiv z_r + i z_i . \tag{6}$$

$$k_1^2 = (2\mu/\hbar^2)(E - U_{1r} - iU_{1i}) \tag{7}$$

k_1 is the wave number inside the nuclear well. a is the radius of the nuclear well. $a = a_0 (A_1^{1/3} + A_2^{1/3})$. A_1 and A_2 are the mass number for the colliding nuclei, respectively. a_0=1.746 fm to give the correct diameter for deuteron (4.4 fm)[9]. C=0.577…is Euler constant. h($k\, a_c$) is related to the logarithmic derivative of Γ function :

$$h(x) = \frac{1}{x^2}\sum_{n=1}^{\infty}\frac{1}{n(n^2 + x^{-2})} - C + \ln(x) \tag{8}$$

When this model was applied to the d+t fusion cross section near 100 keV, it was a surprise to see the good agreement between the theoretical calculation(open circle in Fig.2 and Fig.3) and data points (cross in Fig.2 and

Fig.3) from the evaluated nuclear data file (ENDF/B-VI). There are only two adjustable parameters, U_{1r} and U_{1i}, in this model. We may adjust them to meet the resonance peak (5.01 barns at 110 keV); then, it will reproduce the data points covering the range of energy from 200 eV to 500 keV. Fig.2 depicts the ENDF/B-VI data points , and the results from the selective resonant tunneling model with a square well of U_{1r}= -47.33 MeV, and U_{1i}= -115.25 keV. As a comparison we draw the curve (dashed line) using the empirical formula in the NRL handbook for plasma formulary [10].

$$\sigma = \frac{A_5 + \dfrac{A_2}{(A_4 - A_3E)^2 + 1}}{E[\exp(\dfrac{A_1}{\sqrt{E}}) - 1]} \tag{9}$$

These empirical parameters are evaluated by nonlinear least-squares fitting to available measurement [11]:

$$A_1 = 45.95,\ A_2 = 50200,\ A_3 = 1.368 \times 10^{-2},\ A_4 = 1.076,\ A_5 = 409 \tag{10}$$

We can see in Fig.2 that the results of our calculation agree better than the empirical formula near the resonance peak. In order to show the comparison in the low energy region, Fig.2 is redrawn in the full logarithmic scale in the Fig.3. It is evident, this selective resonant tunneling model with two parameters reproduces the ENDF data points from 10^{-39} barns to 5 barns. It is even better than that of 5 parameter empirical formula in the range of 200 eV to 500 keV. For example, at 200 eV, the empirical formula gives a value less than the ENDF/B-VI data point by 2 orders of magnitude. Since we did not invoke the Taylor expansion to obtain the resonant feature in equation (2), it is possible to have such an expression valid in a wide range of energy. If the p-wave is included in the calculation in addition to the s-wave, the agreement in the range above 500 keV would be further improved.

The selective resonant tunneling model (eq.(2)) is different from the empirical formula (eq.(9)) in calculation of fusion cross section in the following aspects: (1) The selective resonant formula involves two adjustable parameters only(U_{1r} and U_{1i}). In order to fix these two parameters, we need only the experimental value of cross section at the resonance peak, and its position (resonance energy). However, the empirical formula has 5 adjustable parameters (A_1, A_2,…, A_5), we need at least 5 data points to find their values through nonlinear least-squares fitting method; (2) The selective resonant formula is able to estimate the behavior of the cross section at any assumed resonance at low energy (see later in the next section). On the other hand, the empirical formula is not supposed to be applied beyond the range of fitting; (3) Once we have the nuclear potential (U_{1r} and U_{1i}); then, it is possible to estimate where the another resonance may appear nearby (such as the another d+t resonance near 5 MeV, it will be discussed in another paper in parallel with the p+^{11}B resonance near 150 keV and 600 keV). However, there is no information about another resonance in the equation (9).

III. SELECTIVITY IN NUCLEAR REACTION RATE

This surprisingly good agreement with experimental measurements encourages us to further explore the physical meaning involved in this model. Eq.(2) tells that a resonance will appear at W_r=0. However, there is another requirement for W_i as well. W_i must be in the order of (-1) to see a resonance at the point where W_r=0. When W_i approaches zero or much less than (-1), it is hardly to see any resonance peak there [8]. As we mentioned above, this is just the meaning of selectivity in nuclear reaction rate (damping rate), since W_i is directly related to the imaginary part of the nuclear potential well (i.e. the lifetime of penetrating wave). Eq.(4) shows that z_i must be in the order of θ^{-2} to make $W_i \approx$-1. Indeed, the physical meaning of z_i is the ratio of the flight time to lifetime of the penetrating wave inside the nuclear well, because

$$z_i \equiv k_{1i} a = \left| \frac{\mu a}{k_{1r}\hbar^2} U_{1i} \right| = \left| \frac{\dfrac{a}{k_{1r}\hbar/\mu}}{\hbar / U_{1i}} \right| \approx \frac{\tau_{flight}}{\tau_{life}} \approx O(\theta^{-2}) \,. \tag{11}$$

Now lets go back to Fig.1 and see if it could be true to have a jump in cross section at the energy of about 100 eV with neutron emission. At such a low energy, θ^2 >>1; therefore, any jump in cross section is supposed to be induced by a resonance. According to eq.(11), this resonance should select the channel with

$$\tau_{life} = O(\theta^2 \tau_{flight}) >> \tau_{flight} \,. \tag{12}$$

However, the neutron emission happens only in a channel where strong nuclear interaction plays key role, and gives a lifetime in the order of flight time. This channel would never be in resonance with the penetrating projectile at low energy. In this sense, we started to suspect of this unexpected jump in d+t fusion cross section. We checked further the height and the width of this resonance near 100 eV. Fig.4 shows schematically that the peak height of an assumed resonance should be getting higher and higher, and its width should be getting narrower and narrower when the energy of the resonance is getting lower and lower. This behavior can be derived from eq.(2) where the k^2 in the denominator is getting smaller for the lower energy E. The width of the resonance Γ is determined by

$$\Gamma = \left| \frac{(W_i - 1)}{\frac{\partial W_{1r}}{\partial E}} \right|_{resonance} \approx O(\theta^{-2}) \, . \tag{13}$$

This width should become very small when the energy of injected projectile approaches to 100 eV, because the Gamow factor diminishes quickly. However, Fig.1 shows a jump in cross section with a value of almost same as that of 100 keV resonance, and its width is clearly too broad to be consistent with this resonant tunneling model.

Fortunately, in October, 1999, this mistake was corrected by NNDC. This is the first time to correct a piece of experimental data based on the selectivity of selective resonant tunneling model.

IV. IDENTIFICATION OF A LOW ENERGY RESONANCE

Now let us ask a question, if there is a resonance at even lower energy; then, how can we identify this resonance? It must have no neutron emission, or gamma radiation, because the thicker and higher Coulomb barrier will require an even longer lifetime state for such a low energy resonance. The state, which emits neutron or gamma, is a short lifetime state, because the strong interaction or electromagnetic interaction is too strong to have any long lifetime state. Then, the usual nuclear technology for neutron or gamma radiation is no longer applicable to detect this low energy sub-barrier resonance. The calorimetric technology in chemistry turns out to be the better choice, because the energy released in any nuclear reaction is always there. If there is any energetic charged particle as a nuclear product, we may use the nuclear track detector; or we may detect the helium directly. If we are able to identify such kind of low-energy resonant tunneling; then, this is a fusion reaction without strong nuclear radiation.

In conclusion, the nuclear physics for sub-barrier fusion provides a new approach towards nuclear fusion energy with no strong nuclear radiation.

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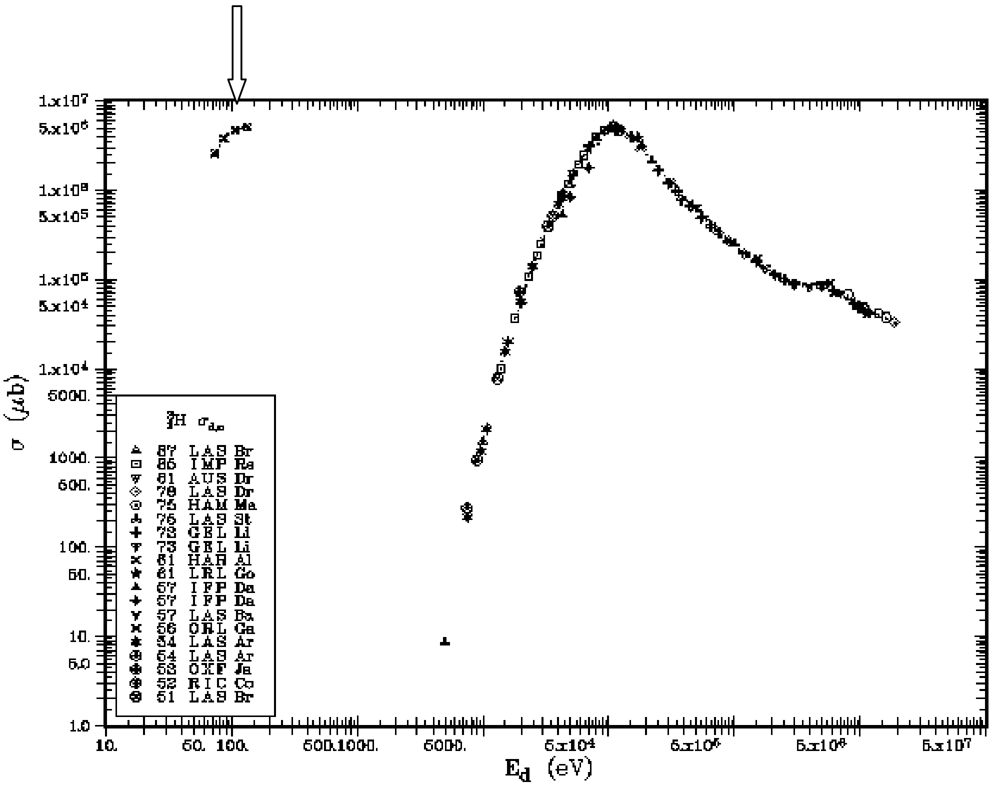
Fig.1



Deuteron Energy in Lab. (eV)

Fig.2

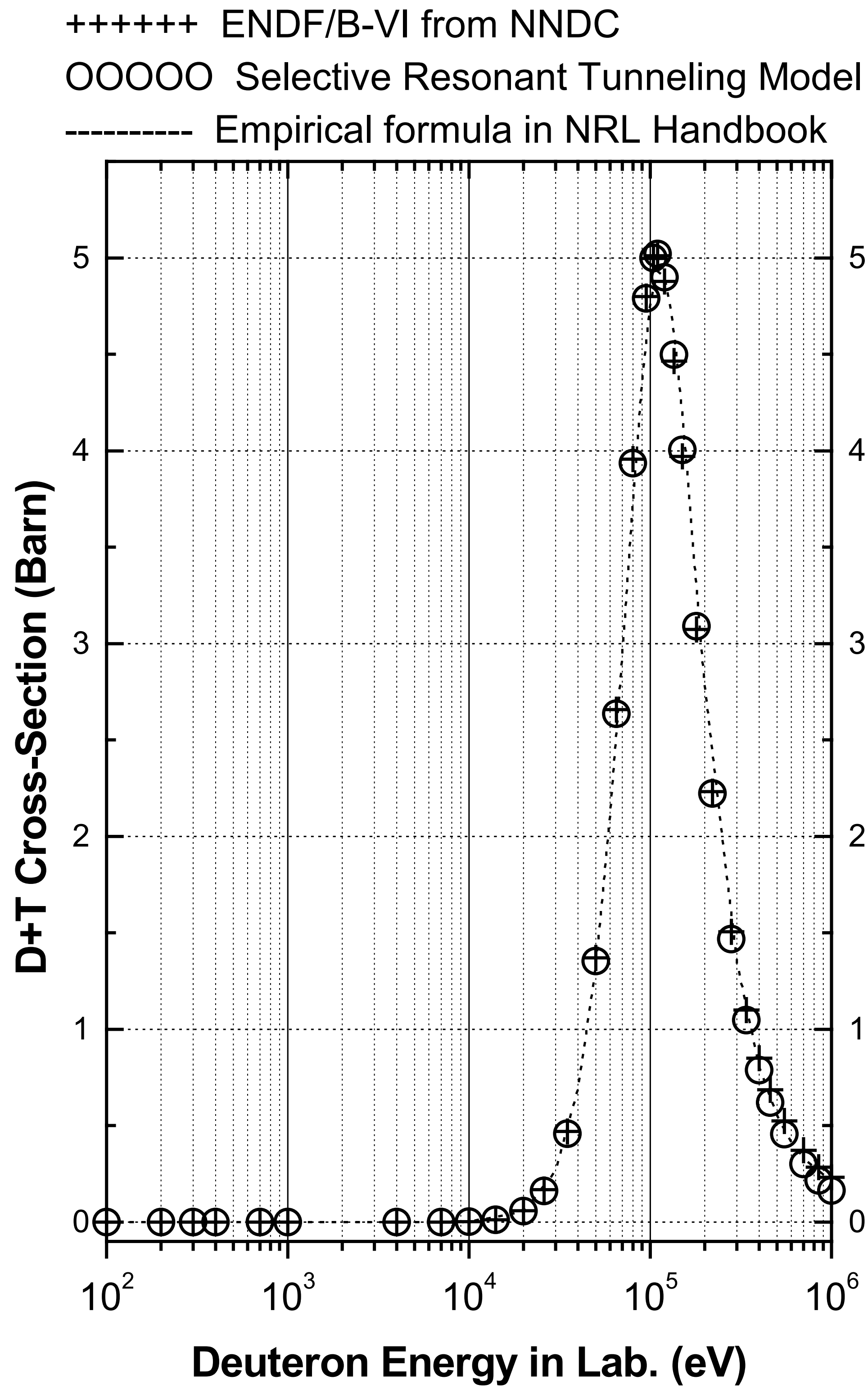
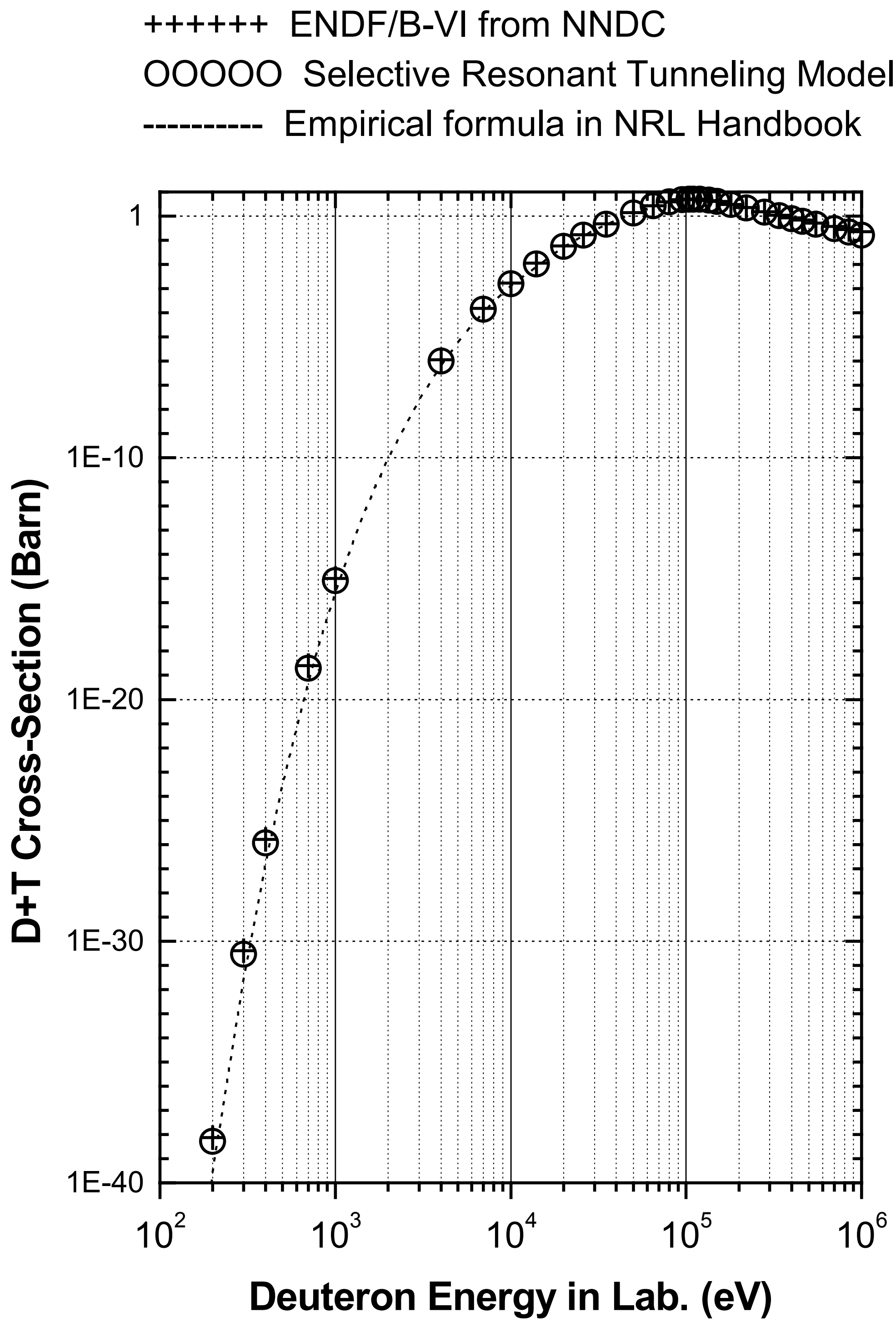
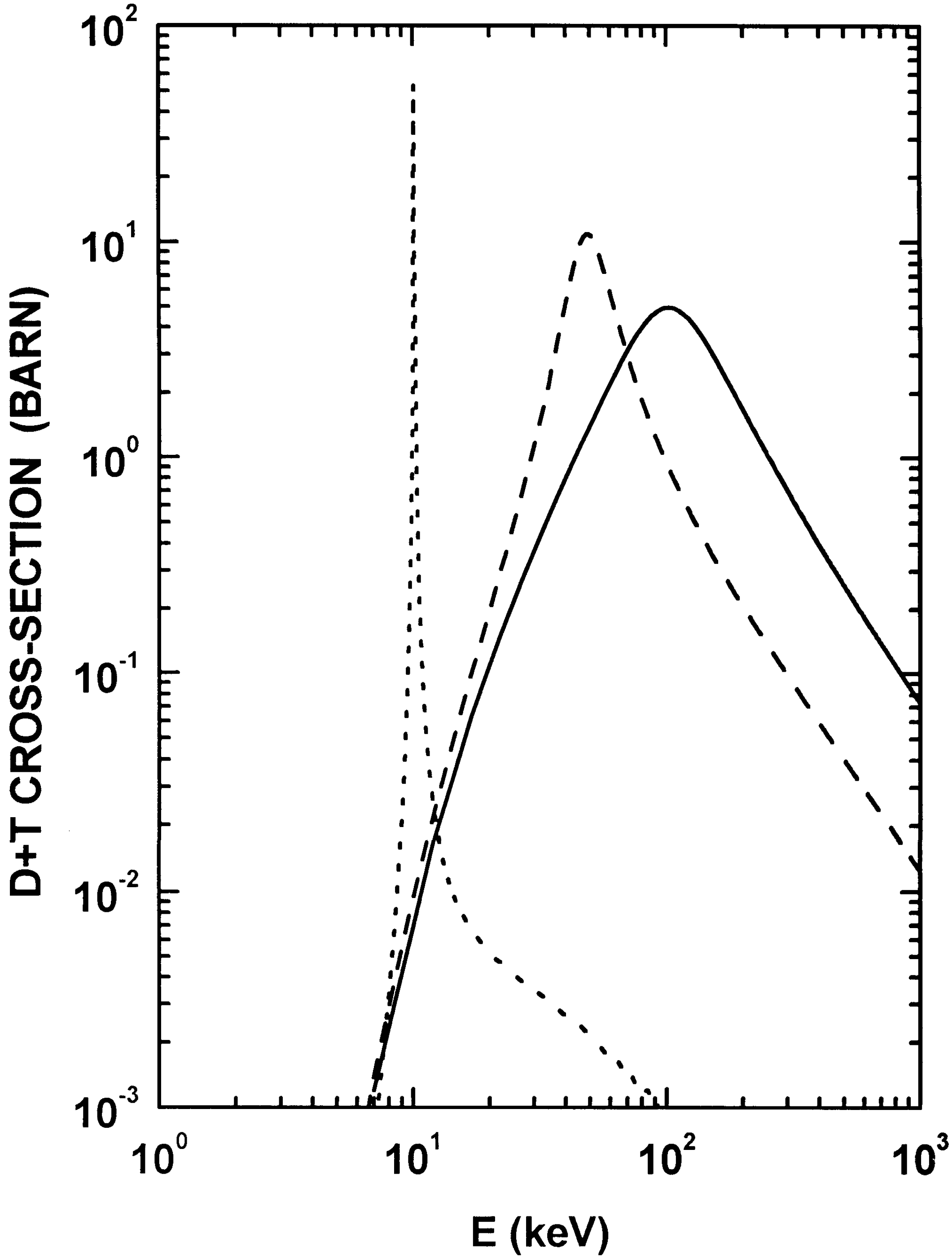

++++++ ENDF/B-VI from NNDC
OOOOO Selective Resonant Tunneling Model
---------- Empirical formula in NRL Handbook
D+T Cross-Section (Barn)
Deuteron Energy in Lab. (eV)
0
1
2
3
4
5
10^2
10^3
10^4
10^5
10^6

Fig.3



++++++ ENDF/B-VI from NNDC
OOOOO Selective Resonant Tunneling Model
---------- Empirical formula in NRL Handbook
D+T Cross-Section (Barn)
1
1E-10
1E-20
1E-30
1E-40
10^2
10^3
10^4
10^5
10^6
Deuteron Energy in Lab. (eV)

Fig.4



D+T CROSS-SECTION (BARN)
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^0
10^1
10^2
10^3
E (keV)

Captions

Fig.1 The deuteron plus triton fusion cross section showing an unexpected jump near 100 eV [1] as pointed by the arrow.

Fig.2 The comparison between the ENDF/B-VI data (cross) and the theoretical calculation based on the selective resonant tunneling model (open circle). The dash line shows the result of 5-parameter empirical formula (Semi-logarithmic scale).

Fig.3 The comparison between the ENDF/B-VI data (cross) and the theoretical calculation based on the selective resonant tunneling model (open circle). The dash line shows the result of 5-parameter empirical formula (Full logarithmic scale).

Fig.4 The shape of d+t fusion cross section predicted by the selective resonant tunneling model for the low energy resonance (if any).

AUTHOR BIOGRAPHY

Xing Zhong Li (PhD(eq.), Tsinghua University, Beijing, China, 1962; PhD, University of Wisconsin (Madison), 1983) is a professor in the department of Physics at the Tsinghua University. In addition, he serves as a member of the Fusion Research Advisory Committee of China, and the head of the Fusion Physics Laboratory at Tsinghua University. His current research interests are fusion physics and industrial plasma engineering.